Solutions to JEE Advanced Home Practice Test -5 |JEE 2024 | Paper-1

Physics

SINGLE CHOICE

1.(C) Here $PA = kx$

$P = \frac{kx}{A}$ as $Ax = V$

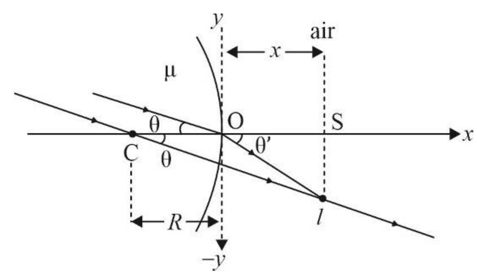
$P = \frac{kV}{A^2}$ $x = \frac{V}{A}$

$PV^{-1} = \text{constant}$

$C = 3R = 3\frac{P_0V_0}{T_0}$

2.(C) Figure shows two rays – one through O and the other through the centre of curvature (C) of the surface. The second ray passes undeviated. The two rays intersect at I.

Snell's law gives



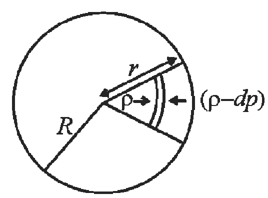
$\sin\theta' = \mu\sin\theta \quad \Rightarrow \theta' = \mu\theta$

$SI = x\tan\theta' = (x+R)\tan\theta \quad \Rightarrow x\theta' = (x+R)\theta$

$\Rightarrow x\mu\theta = (x+R)\theta \quad \Rightarrow (\mu-1)x = R \Rightarrow x = \frac{R}{\mu-1}$

And $SI = (x+R)\theta = R\left(\frac{\mu}{\mu-1}\right)\theta$

3.(B) $(dp\Delta s) = \frac{-G(\Delta s\, dr\,\rho)\times\left(\frac{4}{3}\pi r^3\rho\right)}{r^2}$



$\int_0^P dp = -\int_R^r \frac{4}{3}\pi G\rho^2 r dr$

$\Rightarrow P = \frac{3GM^2}{8\pi R^2}\left(1 - \frac{r^2}{R^4}\right)$

4.(D) $\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T}, \quad T = 2\pi\sqrt{\frac{l}{g}} \approx 2s$

In (A), ΔT for 10 oscillations is 0.1s

Hence, ΔT for 1 oscillation is 0.1/10s $\quad \Rightarrow \frac{\Delta g}{g} = \frac{0.005}{1} + \frac{2(0.01)}{2} = 1.5\%$

In (B & C), $\Delta T = \frac{0.1}{20} = 0.005s \quad \Rightarrow \frac{\Delta g}{g} = \frac{0.005}{1} + \frac{2(0.005)}{2} = 1\%$

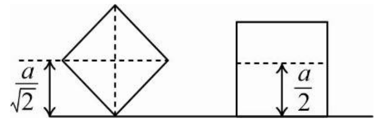
But C is more accurate as amplitude of oscillation is less

In (D), $\Delta T = \frac{0.1}{50} = 0.002s \quad \Rightarrow \frac{\Delta g}{g} = \frac{0.001}{1} + \frac{2(0.002)}{2} = 0.3\%$

Hence, D is most accurate.

COMPREHENSION WITH NUMERICAL TYPE

5.(12.42) $\Delta PE = \Delta KE$



$$mg\left(\frac{a}{\sqrt{2}}-\frac{a}{2}\right)=\frac{1}{2}\left(\frac{Ma^2}{6}+M\left(\frac{a}{\sqrt{2}}\right)^2\right)\omega^2$$

Find $\omega_0 = \sqrt{\dfrac{3g\left(\sqrt{2}-1\right)}{2a}} = \sqrt{\dfrac{30\times 0.414}{1}} = \sqrt{12.42}$

6.(0.25) Use conservation of angular momentum abut B just before and after collision.

$$I_{cm}\omega_0(-\hat{k}) = I_B\omega(-\hat{k}) \quad \Rightarrow \quad \omega = \frac{\omega_0}{4}$$

7.(2) Solid angle formed by a cone having semi vertical angel α is

$$\Omega = 2\pi(1-\cos\alpha) = 2\pi(1-\cos 60°) = \pi$$

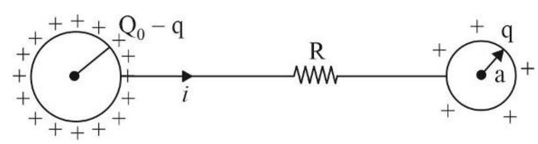
$\therefore$ one fourth of the total number lines emitted from q_1 = Half the number of lines terminating on q_2

$\therefore\ N_1 = 2N_2 \quad \therefore\ |q_1| = 2|q_2|$

8.(90) Half the lines emitted from q_1 terminate on q_2. Remaining half will go to ∞

$$\alpha_{\max} = 90°$$

9.(3) Let the charge on bigger sphere be $(Q_0 - q)$ and that on smaller sphere be q, at time t' after S_2 is closed.



$\frac{dq}{dt} = i =$ current through R

Potential difference between two spheres $= iR$

$$\frac{Q_0-q}{C_1}-\frac{q}{C_2}=iR$$

$$\frac{Q_0}{C_1}-q\left(\frac{C_1+C_2}{C_1C_2}\right)=iR$$

Differentiating, w.r.t. time

$$-\frac{C_1+C_2}{C_1C_2}\frac{dq}{dt}=R\frac{di}{dt}$$

$$\Rightarrow\ -\frac{3}{8\pi\in_0 a}i = R\frac{di}{dt} \qquad \Rightarrow\ \int_{i_0}^{i}\frac{di}{i} = -\frac{3}{8\pi\in_0 aR}\int_0^t dt \qquad \Rightarrow\ \ell n\frac{i}{i_0} = -\frac{3t}{8\pi\in_0 aR}$$

$$\Rightarrow\ i = i_0 e^{-\frac{3t}{(8\pi\in_0 a)R}} = \frac{V}{R}e^{-\frac{3t}{(8\pi\in_0 a)R}} \quad \Rightarrow\ i_0 = \text{current at } t = 0^+ = \frac{V}{R}$$

Rate of change of potential

$$V = \frac{Q-q}{C}; \qquad \frac{dV}{dt} = -\frac{1}{C}\frac{dq}{dt} = -\frac{i}{C}$$

$$\left|\frac{dV}{dt}\right| = \frac{i}{C} = \frac{V}{(8\pi\in_0 a)R}e^{-\frac{3i}{8\pi\in_0 aR}}$$

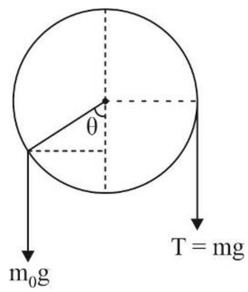
10.(4) Heat dissipated = loss in electrostatic potential energy

$$=\frac{Q_0^2}{2C_1}-\left(\frac{q_1^2}{2C_1}+\frac{q_2^2}{2\cdot\frac{C_1}{2}}\right)=\frac{Q_0^2}{2C_1}-\left[\frac{2Q_0^2}{9C_1}+\frac{Q_0^2}{9C_1}\right]$$

$$=\frac{Q_0^2}{2C_1}-\frac{Q_0^2}{3C_1}=\frac{Q_0^2}{6C_1}=\frac{(C_1V)^2}{6C_1}=\frac{C_1V^2}{6}=\frac{4}{3}\pi\epsilon_0\, aV^2$$

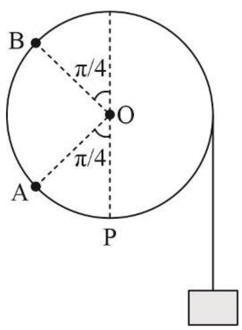
ONE OR MORE THAN ONE CHOICE

11.(AB)



For equilibrium : $m_0gR\sin\theta=mgR$

$$\sqrt{2}m\sin\theta=m\ \Rightarrow\ \theta=\frac{\pi}{4}$$



Between P to A the pulley accelerates as torque due to tension will exceed the torque due to m_0g. Between A to B the pulley system retards and beyond B it will once again accelerate.

For m_0 to climb to the top, we need to ensure that it just manages to cross point B. [Note that in no case m_0 can reach the top point with zero speed]

For kinetic energy of the system to be positive when m_0 reaches B we must have

$$mgR\left(\pi-\frac{\pi}{4}\right)>m_0gR\left(1+\cos\frac{\pi}{4}\right)$$

$$\Rightarrow\ m.\frac{3\pi}{4}>m_0\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\Rightarrow\ m_0<\frac{3\pi}{4\left(\sqrt{2}+1\right)}\left(\sqrt{2}m\right)$$

$$\Rightarrow\ m_0<(0.975)\sqrt{2}m$$

But it is given that $m_0=\sqrt{2}m$

$\therefore\ m_0$ will fail to cross point B.

12.(AC) Let ω be the angular velocity of cylinder at an instant

$$B = \mu_0 \frac{\sigma 2\pi R}{\left(\frac{2\pi}{\omega}\right)} = \mu_0 \sigma R \omega$$

Induced electric field at the location of ring in given by

$$E = \frac{r}{2}\frac{dB}{dt} = \frac{r}{2}\mu_0 \sigma R \frac{d\omega}{dt}$$

For ring, impulse of the torque = change in angular momentum

$$\int qEr\,dt = mr^2\omega^r$$

$$\int q\left(\frac{r\mu_0\sigma R}{2}\frac{d\omega}{dt}\right)r dt = mr^2\omega^r$$

$$\frac{qr^2\mu_0\sigma R\omega_0}{2} = mr^2\omega^r; \qquad \omega^r = \frac{q\mu_0\sigma R\omega_0}{2m}$$

13.(AC) $L\frac{d^2q}{dt^2} + \frac{q}{C} - \frac{CV-q}{C} = 0 \qquad \frac{d^2q}{dt^2} + \frac{2q-CV}{LC} = 0$

$$\Rightarrow \qquad \omega = \sqrt{\frac{2}{LC}} \qquad \Rightarrow \qquad q = \frac{CV}{2}[1-\cos\omega t],\ q' = CV - q = \frac{CV}{2}[1+\cos\omega t]$$

$$I = +\frac{CV\omega}{2}\sin\omega t$$

14.(ABC) Torque about A

$$g \times R = (I\alpha) \quad = (MR^2 + MR^2 + m(R\sqrt{2})^2)\alpha$$

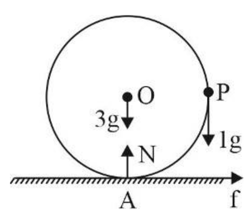
So, $\alpha = \frac{gR}{2MR^2 + 2mR^2} = \frac{g}{2R(M+m)} = \frac{10}{2\times2\times4} = \frac{10}{16} = \frac{5}{8}$

So, acceleration of ring in vertical direction = 0

Acceleration of point mass in vertical direction

As taken from COM frame,

$$4g - N = (1+3)r_{com}\alpha; \qquad N = 4g - 4\left(\frac{1}{2}\right)\left(\frac{5}{8}\right)$$



$$N = 40 - \frac{5}{4}; \quad N = \frac{155}{4}N$$

$$1g\times 2 - f\times 2 = (3.2^2 + 1.2^2)\left(\frac{5}{8}\right)$$

$$20 - 2f = 16\times\frac{5}{8} \Rightarrow f = 5N$$

15.(A) Let $BD = x,\ v = \frac{dx}{dt}; \qquad V_{Ax} = \frac{d}{dt}\left(\frac{x}{2}\right) = \frac{V}{2}$

$$V_{Ay} = \left(\frac{\ell}{2}\sin 60^\circ\right)\left(\frac{d\theta}{dt}\right) = \frac{v}{4} \Rightarrow V_A \neq V$$

16.(BCD) $V = V_o \sin \omega t$; $i = i_o \sin\left(\omega t + \frac{\pi}{3}\right)$

$$P_{\text{in}\,st} = Vi = V_o i_o \sin(\omega t)\sin\left(\omega t + \frac{\pi}{3}\right)$$

(A) $P_{inst}\Big|_{t=\frac{\pi}{2\omega}} = V_o i_o \sin\frac{\pi}{2}\sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right) > 0$

(B) For t between $0 < t < \frac{2\pi}{3\omega}$, both $\sin(\omega t)$ & $\sin\left(\omega t + \frac{\pi}{3}\right)$ terms are positive

(C) For $t = \frac{5\pi}{6\omega}$, $\sin \omega t > 0$, $\sin\left(\omega t + \frac{\pi}{3}\right) < 0.$ Hence positive

(D) If ω is increased slightly, impedance increase and hence, ϕ decreases.

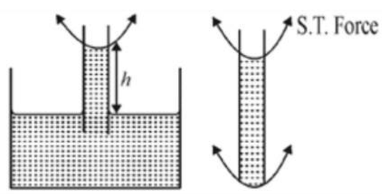
INTEGER TYPE

17.(3) For the excited state, n = 2.

Hence $-13.6eV\left(\frac{1}{6^2} - \frac{1}{2^2}\right)z^2 = 10.20 + 1700 = 27.20eV$

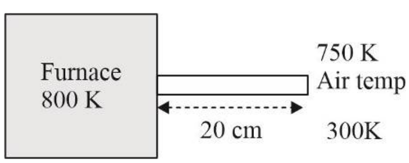
$$-\left(\frac{1}{36} - \frac{1}{4}\right)Z^2 = \frac{27.20}{13.6} \text{or}\, Z = 3$$

18.(2) When the capillary is inside the liquid, the surface tension force supports the weight of liquid of height 'h'



When the capillary is taken out from the liquid similar type of surface tension force acts at the bottom also, as shown in second figure. Hence, now it can support weight of a liquid of height $2h$.

19.(74) Heat flowing through the rod per second in steady state,



$$\frac{dQ}{dt} = \frac{KAd\theta}{x} \qquad \ldots\text{(i)}$$

Heat radiated from the open end of the rod per second in steady state,

$$\frac{dQ}{dt} = A\sigma(T^4 - T_0^4) \qquad \ldots\text{(ii)}$$

From equation (i) and (ii)

$$\frac{Kd\theta}{x} = \sigma(T^4 - T_0^4)$$

$$\frac{K \times 50}{0.2} = 6.0 \times 10^{-8}[(7.5)^4 - (3)^4] \times 10^8 \quad \Rightarrow \quad K = 74\, W/m.K$$

CHEMISTRY

SINGLE CHOICE

1.(B) Because reagent is Li / NH_3 (i.e., $Li^+ + e^-$ and $H - NH_2$] there will be trans addition at both the $\equiv$ bonds

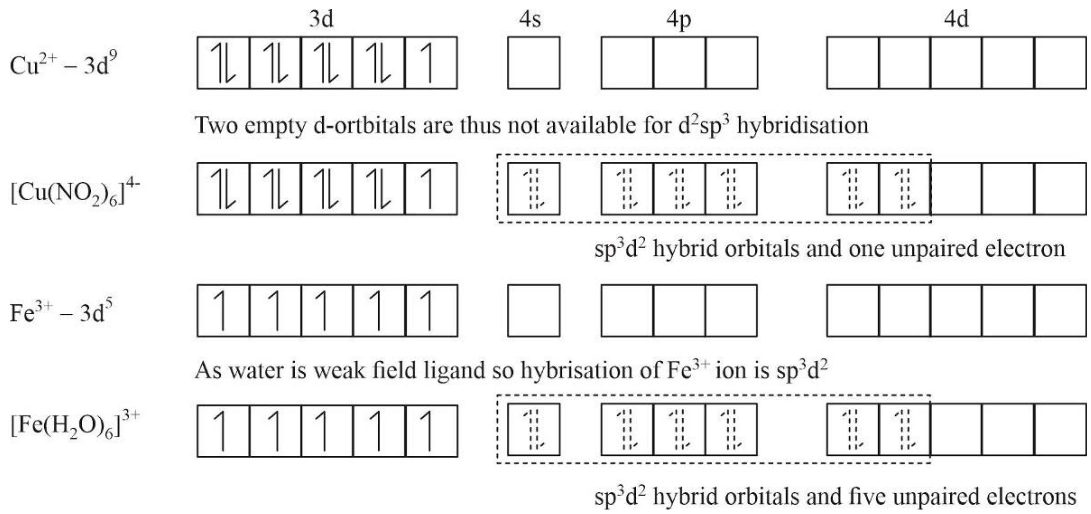
2.(C) Gauche of ethylene glycol is most stable because of intramolecular H-bonding.

3.(A) % packing efficiency $= \dfrac{\text{Area of spheres}}{\text{Area of Hexagon}} \times 100 = \dfrac{3\pi r^2}{\left(\dfrac{\sqrt{3}}{4}a^2\right) \times 6}$

And $a = 2r$

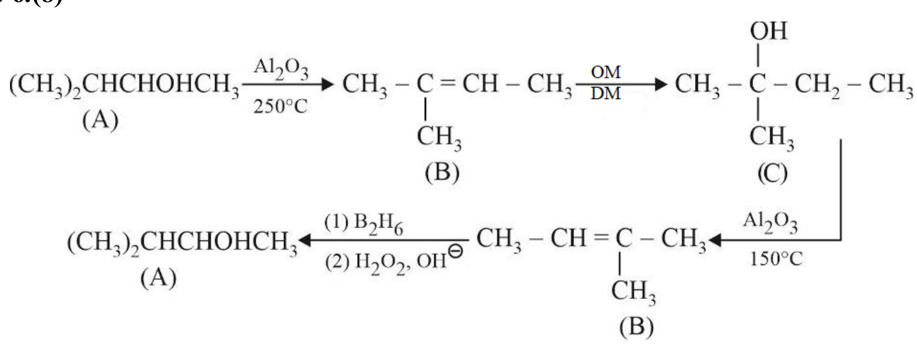
$\therefore$ % packing efficiency = 90.64%

4.(C)



COMPREHENSION WITH NUMERICAL TYPE

5.(5) & 6.(8)

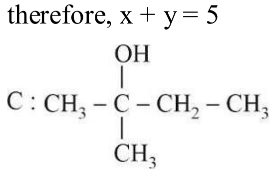


A : $(CH_3)_2CHCHOHCH_3$

3-Methylbutan-2-ol

$x = 3$, $y = 2$

therefore, $x + y = 5$



Number of β hydrogen atoms in the above structure = 8

7.(140) $q = q_{AB} + q_{BC} + q_{CD} + q_{DA}$

$$= -1R \times 300 \ln 2 + 1 \times \frac{5R}{2} \times (400-300) + 1R \times 400 \ln 2 + 1 \times \frac{5R}{2} \times (300-400)$$

($\because q_{AB} = -W_{AB} = -1R \times 300 \ln 2$ since process is reversible isothermal for which $\Delta U = 0$)

($\because q_{BC} = \Delta H_{BC} = 1 \times \frac{5R}{2} \times (400-300)$ since process is reversible isobaric

($\because q_{CD} = -W_{CD} = 1R \times 300 \ln 2$ since process is reversible isothermal for which $\Delta U = 0$)

($\because q_{DA} = \Delta H_{DA} = 1 \times \frac{5R}{2} \times (300-400)$ since process is reversible isobaric)

So, $q = 100R \ln 2 = 140$

8.(500) For process $A \to C$

$\Delta H = \Delta H_{A-B} + \Delta H_{B-C}$

$\Delta H = 0 + \frac{5R}{2}(100)$; $\Delta H = 250R = 500$ cal

9.(500) & 10.(457.14)

$X_A = 0.75$ $X_B = 0.25$

P_{bubble} point $= X_A P_A^0 + X_B P_B^0$

$= X_A P_A^0 + X_B P_B^0$ $= 0.75 \times 400 + 0.25 \times 800 = 500$ mm

$y_A = 0.75$ $y_B = 0.25$

At dew point

$$\frac{1}{P_T} = \frac{y_A}{P_A^0} + \frac{y_B}{P_B^0} \Rightarrow \frac{1}{P_T} = \frac{0.75}{400} + \frac{0.25}{800} = \frac{1.5+0.25}{800} \Rightarrow P_T = \frac{800}{1.75} = 457.14 \text{ mm Hg}$$

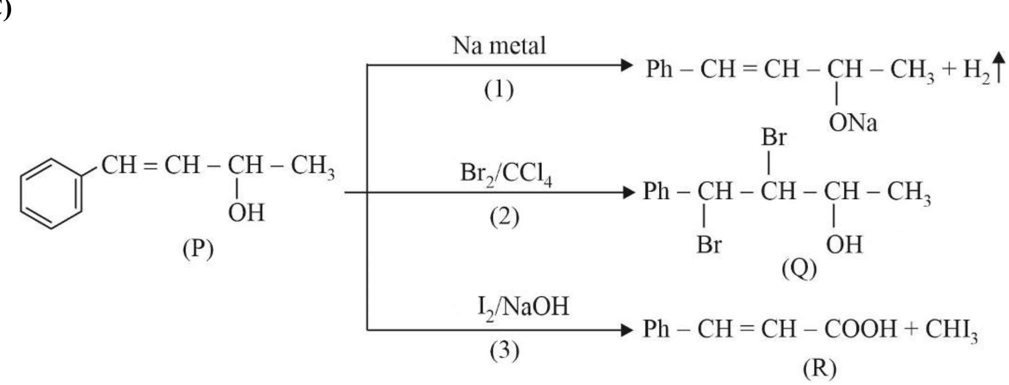
Below dew point only vapour phase exists

ONE OR MORE THAN ONE CHOICE

11.(ABCD) In $\beta-D$ glucopyranose all the substituents occupy the sterically preferred equatorial position making it more stable.

Refer Notes for option B,C & D

12.(BC)



'P' has optical and geometrical isomers.

13.(A)

(A) $\Delta G = \Delta H - T\Delta S < 0$ as $\Delta S < 0$ so ΔH has to be negative

(B) Micelles formation will take place above T_k and above CMC

(C) $Fe(OH)_3$ solution can be prepared by the hydrolysis of $FeCl_3$ solution adsorbs Fe^{3+} and this is positively charged

$$FeCl_3 + 3H_2O \rightleftharpoons Fe(OH)_3 + 3HCl$$

$$Fe(OH)_3 + FeCl_3 \rightarrow \underset{\text{Fixed part}}{Fe(OH)_3, Fe^{3+}}$$

$$\underset{\text{Diffused part}}{3Cl^-}$$

Positive charge on colloidal sol is due to adsorption of Fe^{3+} ion (common ion between $Fe(OH)_3$ and $FeCl_3$)

(D) Fe^{3+} ions will have greater flocculatibility power so smaller flocculating value

14.(ABD) $\Delta S = nC_v \ln\left(\frac{T_f}{T_i}\right) + nR\ln\left(\frac{V_f}{V_i}\right) = 5\ln\frac{373}{298} + 2\ln\frac{10}{1}$

$\Delta H = nC_p\Delta T = 7(75) = 525\,\text{cal}$

$\Delta E = nC_v\Delta T = 5(75) = 375\,\text{cal}$

$\Delta G = \Delta H - \Delta(TS)$

15.(ABD)

(A) $2[Ag(CN)_2]^-(aq) + Zn(s) \rightarrow 2Ag(s) + [Zn(CN)_4]^{2-}(aq)$

Here Zn is used as a reducing agent

(B) Fact

(C) Magnesite $(MgCO_3)$ is ore of magnesium

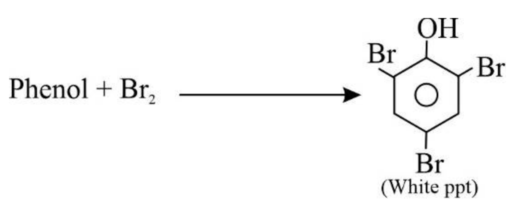
(D) $SnO_2 + 2C \longrightarrow Sn + 2CO$

$2PbS + 3O_2 \longrightarrow 2PbO + 2SO_2$

$PbO + C \longrightarrow Pb + CO$

16.(ACD) $2KBr + MnO_2 + 3H_2SO_4 \longrightarrow 2KHSO_4 + MnSO_4 + 2H_2O + Br_2(X)$

(A)



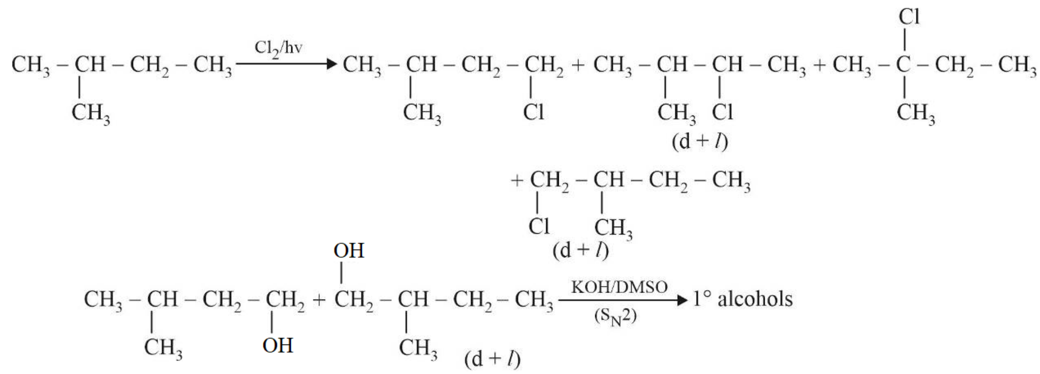
(B) AgBr ppt. (pale yellow) is only partially soluble in NH_4OH

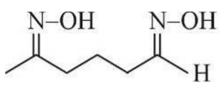
(C) $2KI + Br_2 \longrightarrow 2KBr + I_2$ (violet) soluble in organic layer

(D) $K_2Cr_2O_7 + 6KBr + 7H_2SO_4 \longrightarrow 3Br_2 + Cr_2(SO_4)_3 + 4K_2SO_4 + 7H_2O$

INTEGER TYPE

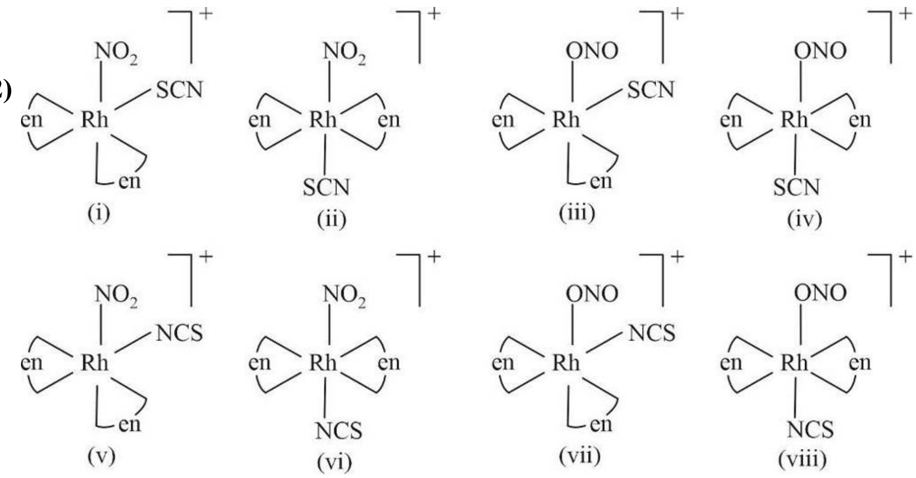
17.(13) x = 4, Y = 6, Z = 3



18.(4) 

Two Geometrical centers ; Total isomers = $2^2 = 4$

19.(12)

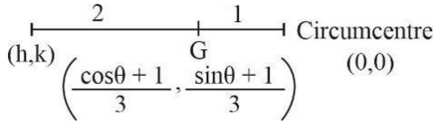


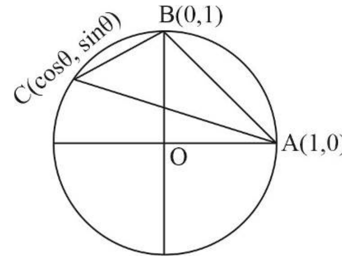
(i), (iii), (v), (vii) will have d & *l* forms

Mathematics

SINGLE CHOICE

1.(A) Let $C(\cos\theta, \sin\theta)$; $H(h,k)$ is the orthocentre of the ΔABC



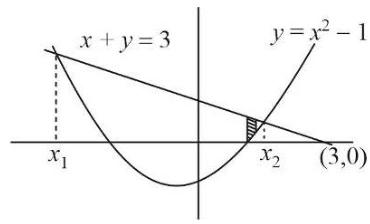


$h = 1 + \cos\theta$; $k = 1 + \sin\theta$

$(x-1)^2 + (y-1)^2 = 1$

$x^2 + y^2 - 2x - 2y + 1 = 0$

2.(D) $3 - x = x^2 - 1 \Rightarrow x^2 + x - 4 = 0$



$x_1 + x_2 = -1$

$x_1 x_2 = -4$ ...(i)

$A = \int_{x_1}^{x_2} [(3-x) - (x^2 - 1)]dx = \int_{x_1}^{x_2} (4 - x - x^2)dx$ use (i)

3.(B) $p = \frac{{}^{13}C_2 \cdot {}^4C_1 \cdot {}^4C_1}{{}^{52}C_2}$ (probability that the cards are higher or lower rank)

$= \frac{16}{17} \Rightarrow$ P(same) $= \frac{1}{17}$

Now, P(H) + P(L) + P(same) = 1

$\therefore$ P(H) = P(L) $= \frac{8}{17}$

4.(A) As α is the fifth non-real root of unity $\therefore\ \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1 = 0$

β is the fourth non real root of unity $\therefore\ \beta^3 + \beta^2 + \beta + 1 = 0$

Consider $(1+\alpha)(1+\alpha^2)(1+\alpha^4)(1+\beta)(1+\beta^2)(1+\beta^3)$

$= (1+\alpha+\alpha^2+\alpha^3)(1+\alpha^4)(1+\beta+\beta^2+\beta^3)(1+\beta^3) = 0$

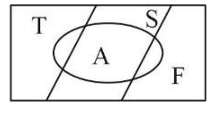
COMPREHENSION WITH NUMERICAL TYPE

5.(0.48) A : She get a success

T : She studies 10 hrs : P(T) = 0.1

S : She studies 7 hrs : P(S) = 0.2

F : She studies 4 hrs : P(F) = 0.7



P(A/T) = 0.8 ; P(A/S) = 0.6 ; P(A/F) = 0.4

$P(A) = P(A \cap T) + P(A \cap S) + P(A \cap F)$

$= P(T) \cdot P(A/T) + P(S) \cdot P(A/S) + P(F) \cdot P(A/F)$

$= (0.1)(0.8) + (0.2)(0.6) + (0.7)(0.4)$

$= 0.08 + 0.12 + 0.28 = 0.48$

6.(0.58) $P(F/A)=\dfrac{P(F\cap A)}{P(A)}=\dfrac{(0.7)(0.4)}{0.48}=\dfrac{0.28}{0.48}=\dfrac{7}{12}=0.5833333$

7.(8) & 8.(10)

$D=\alpha-8$; If $D_1=2(\alpha+\beta-10),\ D_2=5(\beta-2),\ D_3=\beta-2$

9.(12) Given $(4a+3)x-(a+1)y-(2a+1)=0$

$(3x-y-1)+a(4x-y-2)=0$

Family of lines passes through the fixed-point P which is the intersection of

$3x-y=1$ & $4x-y=2$

Solving P(1,2)

Now, we have $y-2=m(x-1)$ ...(i)

This makes an angle of $\pi/4$ with $3x-4y=2$ with slope ¾

$\therefore \left|\dfrac{m-(3/4)}{1+(3m/4)}\right|=1$; $\dfrac{4m-3}{4+3m}=\pm1 \Rightarrow 4m-3=4+4m$ (with +ve sign)

$m=7$

With -ve sign

$4m-3=-4-3m$

$7m=-1 \Rightarrow m=-\dfrac{1}{7}$ (rejected)

Hence the line is

$y-2=7(x-1)$

$7x-y-5=0$

10.(4) Given $(4a+3)x-(a+1)y-(2a+1)=0$

$(3x-y-1)+a(4x-y-2)=0$

Family of lines passes through the fixed-point P which is the intersection of

$3x-y=1$ & $4x-y=2$

Solving P(1,2)

Again $y-2=m(x-1)$

$x=0$; $y=2-m$; $y=0,\ x=1-\dfrac{2}{m}$ $\quad\therefore\ 2A=(2-m)\left(1-\dfrac{2}{m}\right)(m<0)$

$2A=2-m-\dfrac{4}{m}+2=4+\left(-m-\dfrac{4}{m}\right)$

Let $-m=M\quad(M>0)$

$2A=4+M+\dfrac{4}{M}=4+\left(\sqrt{M}-\dfrac{2}{\sqrt{M}}\right)^2+4=8+\left(\sqrt{M}-\dfrac{2}{\sqrt{M}}\right)^2$

Area is minimum if $M=2 \Rightarrow m=-2$

$2A|_{\min}=8 \Rightarrow A|_{\min}=4$

ONE OR MORE THAN ONE CHOICE

11.(ABD) $A^2 = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$

We have $A^2 - 4A - 5I_3$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0$$

$\Rightarrow \; 5I_3 = A^2 - 4A = A(A - 4I_3)$

$\Rightarrow \; I_3 = A\left[\frac{1}{5}(A - 4I_3)\right] \Rightarrow A^{-1} = \frac{1}{5}(A - 4I_3)$

Note that $|A| = 5$. Since $|A^3| = |A|^3 = 5^3 \neq 0,\ A^3$ is invertible

Similarly, A^2 is invertible

12.(ABD) Slope of y tangent at $x = 0$ is $\tan^{-1}(2) \;\Rightarrow$ (A) is correct

f is obviously increasing for (0,3)

$(f'(x) \geq 0) \;\Rightarrow$ (B) is correct

$x = 1$ being an inflection point

$x = 1$ is inflection but not as extremum as $f'(x)$ does not change sign hence (C) is not correct

13.(ABCD) (B) $1 \geq P(A) + P(B) - P(A \cap B)$ or $P(A \cup B) \leq 1 \Rightarrow (B)$

(C) Let $P(A) > P(A/B)$

or $P(A) > \dfrac{P(A \cap B)}{P(B)}$

$P(A) \cdot P(B) > P(A \cap B)$ …(i)

TPT $P(A/B^C) > P(A)$

$\dfrac{P(A \cap B^C)}{P(B^C)} > P(A)$

$P(A) - P(A \cap B) > P(A)[1 - P(B)]$

$-P(A \cap B) > -P(\cap B) > -P(A) \cdot P(B)$

or $P(A) \cdot P(B) > P(A \cap B)$ …(ii)

from (i) and (ii)

$P(A) > P(A/B) \Rightarrow P(A/B^C) > P(A)$

14.(AC) For $|AB| = 0 \;\Rightarrow |A| \cdot |B| = 0 \;\Rightarrow |A| \neq 0, |B| = 0$

$AA^{-1} = I \;\Rightarrow\; |A| \cdot |A|^{-1} = |I| = 1 \;\Rightarrow\; |A^{-1}| = \dfrac{1}{|A|} = |A|^{-1}$

15.(BCD) We have $S_n = x + x^4 + x^7 + x^{10} + ...$ (2n terms)

$C_n = x^2 + x^5 + x^8 + x^{11} + ...$ (2n terms)

$T_n = x^3 + x^6 + x^9 + x^{12} + ...$ (2n terms)

(B) Clearly $S_n > C_n > T_n$ as x is a proper fraction $\therefore\ x > x^2 > x^3$ & so on

(C) $\lim_{n\to\infty}(S_n + C_n + T_n) = x + x^2 + x^3 + ...\infty = \frac{x}{1-x}$

(D) We have $S_n = C_n + T_n \Rightarrow x\frac{\left((x^3)^{2n}-1\right)}{(x^3-1)} = x^2\frac{\left((x^3)^{2n}-1\right)}{(x^3-1)} + x^3\frac{\left((x^3)^{2n}-1\right)}{(x^3-1)}$

But $x \neq 1$, as $x \in \left(0, \frac{\pi}{4}\right)$, so we get $x = x^2 + x^3$

$\Rightarrow\ x^2 + x - 1 = 0\ (x \neq 0) \Rightarrow x = \frac{\sqrt{5}-1}{2} = 2\sin\frac{\pi}{10} \in \left(0, \frac{\pi}{4}\right)$

16.(BC) Given $\omega = \frac{(1-z)^2}{1-z^2} = \frac{1-z}{1+z} = \frac{z\bar{z}-z}{z\bar{z}+z} = \frac{\bar{z}-1}{\bar{z}+1} = -\left(\overline{\frac{1-z}{1+z}}\right) = -\bar{\omega}$ (As $|z| = 1 \Rightarrow z\bar{z} = 1$)

$\therefore\ \omega + \bar{\omega} = 0 \Rightarrow \omega$ is purely imaginary. Hence ω lies on y-axis

(A) z lies on perpendicular bisector of (2,4) and (2,-4) which is x-axis

(B) z lies on perpendicular bisector of (3,-4) and (-3,-4) which is y-axis

(C) z lies on perpendicular bisector of (-2,0) and (2,0) which is y-axis

(D) z lies on either of the rays emanating from (0,1) and (0,-1) towards $+\infty$ & $-\propto$ respectively along y axis not complete y-axis

INTEGER TYPE

17.(4) As $P(x)$ is an odd function

Hence $P(-x) = -P(x) \Rightarrow P(-3) = -P(3) = -6$

Let $P(x) = Q(x^2 - 9) + ax + b$ (where Q is quotient and $(ax+b) = g(x)$ = remainder)

Now $P(3) = 3a + b = 6$ …(i)

$P(-3) = -3a + b = -6$ …(ii)

Hence $b = 0$ & $a = 2$

Hence $g(x) = 2x \Rightarrow g(2) = 4$

18.(3) We have $\frac{\sin A}{c\sin B} = \frac{a}{bc}$ $\quad\therefore\ \frac{\sin B}{c} + \frac{\sin C}{b} = \frac{c}{ab} + \frac{b}{ac}$

$\Rightarrow\ \frac{b\sin B + c\sin C}{bc} = \frac{c^2+b^2}{abc} \Rightarrow a = \frac{b^2+c^2}{b\sin B + c\sin C} = \frac{b(2R\sin B) + c(2R\sin C)}{b\sin B + c\sin C}$

$\Rightarrow\ a = 2R \quad \left(As\ \frac{b}{\sin B} = \frac{c}{\sin C} = \frac{a}{\sin A} = 2R\right)$; Hence $\angle A = \frac{\pi}{2}$

19.(7) The required area $= \frac{1}{2}\left|\overrightarrow{BE}\times\overrightarrow{DE} + \overrightarrow{EC}\times\overrightarrow{DE}\right| = \frac{1}{2}\left|\overrightarrow{BC}\times\overrightarrow{DE}\right|$

$= \frac{1}{2}\left|\left(\overrightarrow{AC} - \overrightarrow{AB}\right)\times\overrightarrow{DE}\right| = \frac{1}{2}\left|\left(-\hat{i} + 4\hat{j}\right)\times(4\hat{i} - 2\hat{j})\right| = \frac{1}{2}|2 - 16| = 7$ (square unit)